Exercise 16

Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval.

$$g(x) = \frac{x - 1}{3x + 6}, \quad (-\infty, -2)$$

Solution

Assuming that $-\infty < a < -2$, calculate the limit of $g(x)$ as x approaches a using the limit laws.

$$
\begin{aligned}
\lim_{x \to a} g(x) &= \lim_{x \to a} \frac{x - 1}{3x + 6} \\[2mm]
&= \frac{\displaystyle\lim_{x \to a} (x - 1)}{\displaystyle\lim_{x \to a} (3x + 6)} \\[2mm]
&= \frac{\displaystyle\lim_{x \to a} x - \lim_{x \to a} 1}{\displaystyle\lim_{x \to a} 3x + \lim_{x \to a} 6} \\[2mm]
&= \frac{\displaystyle\lim_{x \to a} x - \lim_{x \to a} 1}{\displaystyle 3\lim_{x \to a} x + \lim_{x \to a} 6} \\[2mm]
&= \frac{a - 1}{3a + 6} \\[2mm]
&= g(a)
\end{aligned}
$$

Therefore, $g(x) = \frac{x-1}{3x+6}$ is continuous on the interval $(-\infty, -2)$.